

Roll No.

1982

**B. E. / B. Tech. 1st Semester
Examination – December, 2013**

MATHEMATICS-I

'E' Scheme

Paper : MATH-101(E)

Time : Three hours]

[Maximum Marks : 100

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, taking at least *two* questions from each Part. All questions carry equal marks.

PART – A

1. (a) Discuss the convergence of the series :

$$\frac{1}{2} + \frac{2}{3}x + \left(\frac{3}{4}\right)^2 \cdot x^2 + \left(\frac{4}{5}\right)^3 x^3 + \dots \infty$$

- (b) Discuss the convergence of the series :

$$\frac{1}{6}x^2 - \frac{2}{11}x^3 + \frac{3}{16}x^4 - \frac{4}{21}x^5 + \frac{5}{26}x^6 \dots \dots \dots$$

2. (a) Test the convergence of the series :

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots (-1)^{n-1} \frac{x^n}{n} + \dots \infty$$

- (b) Discuss the maxima and minima of :

$$\sin x + \sin y + \sin(x+y)$$

3. (a) Expand $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$. Hence find the value of $\sin 91^\circ$.

- (b) Show that for the parabola $y^2 = 4ax$, ρ^2 varies as $(SP)^3$, where ρ is the radius of curvature at any point P of the parabola and S is the focus of the parabola.

4. (a) If $x + y = 2e^\theta \cos \phi$, show that :

$$\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial \phi^2} = 4xy \frac{\partial^2 u}{\partial x \partial y}$$

- (b) If $u = \operatorname{cosec}^{-1} \left(\frac{(x)^{\frac{1}{2}} + (y)^{\frac{1}{2}}}{\frac{1}{(x)^{\frac{1}{3}} + (y)^{\frac{1}{3}}}} \right)^{\frac{1}{2}}$, then prove that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} = \frac{\tan u}{12} \left(\frac{13}{12} + \frac{\tan^2 u}{12} \right)$$

PART - B

5. (a) Change the order of integration in

$$I = \int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx \text{ and hence evaluate the same.}$$

- (b) Show by double integration, that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$.

6. (a) Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

- (b) Prove that :

$$\Gamma(m) \Gamma(m + 1/2) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m)$$

7. (a) Find the directional derivatives of the function $f = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4).

- (b) Define curl of a vector point function and give its geometrical interpretation, also prove that $\text{curl curl } F = \text{grad } (\text{div } F) - \nabla^2 F$.

8. (a) Verify divergence theorem for $\vec{F} = 4xi - 2y^2j + z^2k$ taken over the region bounded by $x^2 + y^2 = 4, z = 0, z = 3$.

(b) Apply Green's theorem to evaluate :

$$\oint_C \left[(3x^2 - 8y^2) dx + (4y - 6xy) dy \right]$$

where C is the boundary of the region defined by

the $x = 0, y = 0, x + y = 1$

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